A Generalized Model to Predict Minimum Particle Transport Velocities in Multiphase Air–Water Horizontal Pipes

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DOI 10.1002/aic.14824
Published online May 4, 2015 in Wiley Online Library (wileyonlinelibrary.com)

A new model is proposed to predict minimum flow rates required to constantly move particles in both intermittent and stratified flow regimes. The new model consists of a single-phase flow model along with an appropriate length scale to be extended to multiphase flow regime. A comparison of the new model with experimental data in a multiphase air–water flow shows that the new model is able to predict minimum flow rates well for a wide range of operating conditions. The new model can capture the effects of particle size, particle concentration, and pipe size as confirmed by experimental data. A comparison of the new model with previously proposed models in the literature shows that the new model improves critical velocity predictions significantly. Moreover, the new model is the only model that takes into account the effect of particle concentration and can predict critical velocity in both intermittent and stratified flow regimes. © 2015 American Institute of Chemical Engineers *AIChE J*, 61: 2634–2646, 2015
Keywords: particulate flows, particle transport model, multiphase flow, stratified flow, intermittent flow, low liquid loading flow, flow assurance

Introduction

Particle transport in conduits is an active research area. Different industries are interested in transporting particles through or out of their respective pipelines/systems. The removal of particles from a pipeline can be achieved at times simply by running the pump at full capacity, however, this solution may lead to high operation costs and more importantly erosion resulting in facility integrity issues. Therefore, it is crucial to find the velocity to transport particles economically and reduce potential damage to the pipeline if particles are erosive. When particles become trapped inside a pipeline, it can lead to three major problems: blockage, erosion, and corrosion. First, if the velocity is low enough, particles will begin to collect into what are called beds or dunes. These beds or dunes can continue to collect particles unless the fluid velocity within the pipe is increased. Overtime, if the bed or dune is not removed, it can partially block the cross-sectional area of the pipe causing higher than expected pressure drops. The second problem, which is erosion, can occur in the presence of particles such as sand. At high fluid velocities, sand particles can become fully suspended within the moving fluid. It would seem that the sand transportation problem is solved, until the particles impact the inside surface of the pipe. When the sand is entrained at such high speeds, it can damage the inside of the pipeline and cause the pipeline to fail much sooner than desired.[1] Lastly, corrosion can be an issue when sand is present in a pipeline. Many companies inject chemicals into the pipeline to protect them from corrosion failures. When sand is present in a pipe forming beds or dunes, the chemicals might not be able to reach the inner surface area of the pipeline beneath the sand and under deposit corrosion may occur and cause pipe integrity issues requiring unplanned replacements.

Although the discussion in the previous paragraph focuses on liquid-solid transport issues, similar issues, and arguments apply when the transport media contains more than one phase. In multiphase particle transport, particles are usually carried within a phase while other phases affect the carrier phase. For example, particles may be entrained and traveling in liquid phase at the bottom of a horizontal pipe with gas flowing above the liquid phase. The distribution of the liquid and gas phases depends on the operating conditions, and various flow regimes (stratified, intermittent, annular etc.) can be observed in the pipe. While all these flow regimes have different characteristics, the ultimate goal when particles are present in these multiphase systems is their effective transport in the pipe.

In this study, a new multiphase flow model to predict minimum flow rates required to transport particles successfully in intermittent and stratified flow regimes is proposed. A comparison of the new model with experimental data

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available in the literature shows that the new model is able to predict minimum liquid and gas flow rates (critical velocity) well for a wide range of operating conditions.

Literature Review

There are numerous studies that focus on understanding and modeling particle transport in single-phase flow. However, despite its industrial significance, particle transport in multiphase flow received limited attention from the research community. There is a relatively limited number of studies for particle transport in multiphase flow as compared with single-phase flow. The complexity of the phenomena and various physical parameters involved may contribute to this. The existing experimental studies cover a narrow range of operating conditions[2–4] in either intermittent or stratified flow regimes. Additionally, the proposed models, which are tuned to existing data, do not follow the same trends for a wide range of operating conditions. More recently, the authors experimentally investigated particle transport in multiphase flow (air–water) for a wide range of operating conditions[5,6] focusing on the effects of particle concentration and size, pipe size, and multiphase flow regime on particle transport. To the best knowledge of authors, these studies cover the widest range of operating conditions reported in the literature regarding particle transport in multiphase flows.

One of the earliest models developed to predict critical velocity for particle transport in multiphase flow was proposed by Holte et al.[7] They modified Wick's[8] model using experimental data to extend it to stratified air–water flow. Wicks developed a correlation based on experimental data to estimate particle hold up assuming that a stationary sand bed is already formed, and a sand particle is in contact with at least three other particles in a tripod type of support. In 1989, Angelsen et al.[9] modified Wick's[8] model to account for the particle diameter on particle settling characteristics and developed a correlation based on experimental data. They defined critical velocity as the minimum liquid velocity, which prevents a stationary sand bed from forming. They extended their model to multiphase stratified flow using liquid hold up to calculate hydraulic pipe diameter and actual liquid velocity. They concluded that using actual liquid velocity as full pipe velocity and hydraulic pipe diameter as pipe diameter, their single-phase flow model (Wick's modified model) can be applied to multiphase stratified flow. Oudeman[2] proposed a multiphase flow model for large particle hold up by investigating the obtained experimental data in his study and introducing two dimensionless parameters called sand transport rate and liquid flow rate. He suggested sand transport rate to be a power law function of liquid flow rate and obtained the empirical constants in his model experimentally. A couple of years later, Salama[10] proposed a model based on similar parameters that had been used in previous studies. He compared Wick's model with models of Davies[11] and Oroskar and Turian[12] to propose an equation based on similar parameters including empirical constants. To obtain the empirical constants in his model, he used data from DNV Corroline software. Gillies et al.[3] proposed a multiphase flow model by modifying the previous model of Meyer-Peter and Muller.[13] They also performed experiments for high particle concentrations in 0.05-m pipe with a stationary bed already in the pipe. King et al.[14] used Thomas'[15] model for single-phase flow to predict minimum transport pressure drop in multiphase flow. They used frictional velocity equations proposed by Thomas[15] to calculate frictional pressure drop. They compared the actual pressure drop to the calculated frictional pressure drop and concluded that the solids will be transported if the actual pressure drop is higher. In 2001 and 2002, Stevenson et al.[16] and Stevenson and Thorpe[17] experimentally studied the behavior of isolated grains of particles in intermittent and stratified flows. They developed two models to predict the velocity required to prevent a stationary bed from forming in both flow regimes. Particle velocity was measured by measuring the time required to transit a distance. Then, velocity of an isolated particle was correlated to dimensionless groups. To obtain the minimum velocity to ensure particle transport, the correlation was extrapolated to zero particle velocity. In 2007, Danielson[18] modified his single-phase flow model using a drift flux model and extended it to multiphase flow. The model assumes that there is a slip between the liquid velocity and particle velocity, and calculates liquid velocity and particle hold up using OLGA™ 2000 software by modeling the solid phase as a pseudophase with a slip velocity. He reported good agreement between his model and experimental data. Recently, Ibarra et al.[4] performed experiments in a 0.1 m diameter pipe to study critical particle deposition velocity in multiphase stratified flow in horizontal pipes for a limited range of operating conditions. They proposed a model based on combining the Oroskar and Turian[12] model for single-phase flow with the Salama[10] model for two-phase flow. They modified the combined correlation using their experimental data and reported that the use of this equation along with a modified Chisholm liquid hold-up correlation can predict the transition velocity between a moving and stationary bed. To the best of our knowledge, this model is the first model that takes into account the effect of particle concentration on critical particle deposition velocity in multiphase flow.

A review of all the models that have been proposed so far shows that they are not general enough to be applied to a wide range of operating conditions. They are developed either for intermittent or stratified flow regimes or have been proposed using experimental data for a limited range of operating conditions. Also, except for the Ibarra model, they are all independent of particle concentration. In this study, for the first time, a multiphase flow model is proposed that can be applied to both stratified and intermittent flow regimes and also accounts for the effect of particle concentration on critical velocity.

Model Development

The proposed particle transport model in multiphase flow in the current study is based on using a single-phase particle transport model (the Oroskar and Turian model[12] as will be discussed later) and actual liquid velocity. The difference between actual liquid velocity in single-phase and multiphase flow comes from the fact that in multiphase flow, the cross section of the pipe is not fully occupied by liquid. As a result, actual liquid velocity in multiphase flow is calculated by dividing the superficial liquid velocity by liquid hold up

$$V_L = \frac{V_{sl}}{H_L} \quad (1)$$

There are multiphase flow models developed to predict liquid hold up in multiphase flow pipelines. Therefore, the multiphase flow models used in this study to calculate liquid hold up should not be confused by the ultimate goal of this study which is developing a particle transport model in multiphase flow (air–water) flows.

The Oroskar and Turian[12] model has been widely used to predict critical velocity in single-phase flow in horizontal pipes. This model is given in Eq. 2

$$\frac{V_C}{\sqrt{g\ d_p\left(\frac{\rho_p}{\rho_L}-1\right)}}=1.85\ C^{0.1536}(1-C)^{0.3564}\left(\frac{d_p}{D}\right)^{-0.378}{N_{Re}}^{0.09}x^{0.3} \tag{2}$$

where d_p, ρ_p, ρ_L, C, and D are particle diameter, particle and liquid density, particle concentration, and pipe diameter, respectively. V_c is the single-phase critical velocity and N_{Re} is defined as

$$N_{Re}=\frac{D\ \rho_L\sqrt{g\ d_p\left(\frac{\rho_p}{\rho_L}-1\right)}}{\mu_L} \tag{3}$$

In Eq. 2, x is the fraction of eddies that keep particles suspended in the flow. Based on the comparison of experimental data with the model, Oroskar and Turian[12] reported that $x=1$.

This model assumes particles are dispersed by turbulence and calculates critical velocity by balancing the energy required for particle suspension and energy produced from turbulent eddies. In their paper, critical velocity is defined as "the minimum velocity which demarcates flows in which the solids form a bed at the bottom of the pipe from fully suspended flows." In other words, their definition for critical velocity falls between moving bed and dispersed particle flow regimes. In the literature, this flow regime of particles is usually referred to as heterogeneous.[19] A concentration gradient of particles across the pipe occurs due to gravity with some particles entrained in the flow and some moving along the bottom of the pipe.

The proposed model for particle transport in multiphase flow in this study assumes that the particles are transported by the liquid phase as all our experimental studies revealed particles were transported in the liquid phase. Hence, the actual liquid velocity is the determining factor for particle transport in liquid–gas flows. The gas velocity indirectly affects particle transport by changing the liquid velocity. As discussed in the Literature Review section, this concept has been used before by others but with limited success when applied to a broad range of conditions. There are two reasons which may explain why previous studies that tried to extend a single-phase flow model to multiphase flow were not successful. We believe that the most important reason is the single-phase flow model used in these studies, which was either not robust enough or was not compatible with the multiphase experimental data to which the multiphase phase model was compared. The other reason is deciding the appropriate length scale. In multiphase flow, there are generally three common length scales that are used to scale a single-phase flow model to multiphase flow: pipe diameter (D), hydraulic pipe diameter (D_H), and liquid height (L_H). We believe using an appropriate single-phase flow model along with a compatible length scale will result in an accurate multiphase particle transport model. Two of those three length scales require accurate measurement or calculation of liquid hold up.

The proposed model to predict particle critical velocity in multiphase flow in this study uses the Oroskar and Turian

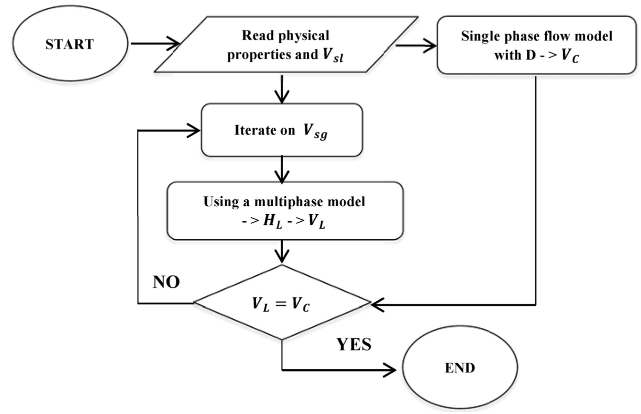

Figure 1. The algorithm used to calculate the critical velocity using pipe diameter as a length scale.

single-phase flow model along with an appropriate multiphase flow model to calculate liquid film velocity. For the intermittent flow regime, Zhang's[20] model is used to calculate liquid film velocity at the bottom of the pipe. For stratified flow, Fan's[21] model is used to calculate liquid hold up and as a result actual liquid film velocity. The reason why the above models were selected to develop a multiphase flow model for particle transport will be discussed later. Appendices A and B contain the governing equations for the models and illustrate how liquid hold up can be calculated using these two models. First, we discuss the algorithm (Figure 1) our model uses to compute the critical velocity. As shown in Figure 1, given physical properties (solid properties including particle concentration, particle size, and particle density, liquid properties including liquid density and liquid viscosity along with pipe diameter) are used by the single-phase flow model (the Oroskar and Turian model) to calculate the required liquid velocity. For the multiphase flow part, using the physical properties and a constant superficial liquid velocity, the model assumes a constant value for superficial gas velocity. Knowing superficial liquid and gas velocities, the Taitel–Dukler model[22] is used to predict the multiphase flow regime. If the Taitel–Dukler model[22] predicts that the combination superficial liquid and gas velocities assumed in the beginning fall in intermittent flow regime, Zhang's model is used to calculate actual liquid film velocity. However, if the assumed superficial liquid and gas velocities fall in stratified flow, Fan's model is used to calculate liquid hold up and correspondingly actual liquid film velocity. Finally, the actual liquid velocity from the multiphase flow part of the code is compared with minimum required velocity from the single-phase flow part obtained using the Oroskar and Turian model. If actual liquid velocity is equal to single-phase flow velocity, assumed values for superficial liquid and gas velocities represent critical velocity values in multiphase flow, otherwise the procedure is repeated with a new value for superficial gas velocity. While in the current approach pipe diameter is used as the length scale, the possibility of using hydraulic pipe diameter (D_H) and liquid height (L_H) to extend the Oroskar and Turian model to multiphase flow regime will be discussed later.

The reason that the Oroskar and Turian single-phase flow model is selected for extension to the multiphase flow region is that previous single-phase flow experimental studies show that this model is able to predict critical velocity well when the carrier liquid is water.[23,24] The Zhang model is used to

Table 1. Operating Conditions of the Experimental Studies

Particle Diameter	0.05 m Diameter Pipe		0.1 m Diameter Pipe	
	$C = 0.01\%$	$C = 0.1\%$	$C = 0.01\%$	$C = 0.1\%$
20 µm 150 µm 300 µm	$V_{sl} \approx 0.011–0.42$ m/s $V_{sg} \approx 0.1–11.23$ m/s		$V_{sl} \approx 0.004–0.43$ m/s $V_{sg} \approx 0.4–12.7$ m/s	

calculate liquid film velocity in intermittent flow regime, as it has been reported that there is good agreement between this model and experimental data[25] and this model is able to calculate liquid film velocity in three-phase flow (air–water–oil) for future studies. Fan's model was developed to predict liquid hold up in the stratified flow regime and more specifically the low liquid loading region. As the experimental data that the new model is compared with includes the low liquid loading region and existing multiphase flow models deviate a lot in this region,[21] Fan's model is used to calculate liquid hold up and liquid film velocity in stratified flow.

Results and Discussions

To validate the accuracy of the proposed model, a consistent set of multiphase flow experimental data is needed. By consistent, we mean the definition of critical velocity at which the multiphase experimental data is obtained should be sufficiently close to the definition of the single-flow model being used. For example, extending a single-phase flow model that defines critical velocity as pick up velocity to the multiphase flow regime and comparing the extended multiphase flow model with experimental data obtained using dispersed particle flow as critical velocity is not reasonable. As aforementioned, Oroskar and Turian, in their study, defined critical velocity as the velocity that is higher than that necessary for a moving bed and lower than that required to achieve dispersed flow. While this definition may not provide a singular result, it clearly defines lower and upper limits.

Recently, a series of experimental data have been reported regarding particle transport in multiphase flow that defines critical velocity within the lower and upper limits defined by Oroskar and Turian. The authors[5] performed experiments in air–water two-phase flow in both intermittent and stratified flow regimes and reported experimental data in 0.05- and 0.1-m pipe diameters. They investigated the effect of particle size, particle concentration, and pipe size for low particle concentrations. In another study, the authors[6,26] investigated the same physical parameters in the low liquid flow rates region and reported that none of the existing multiphase flow models are able to predict critical velocity within the range of very high superficial gas and very low superficial liquid velocities. In both studies, critical velocity is defined as the minimum superficial liquid and gas velocities required to constantly move particles along the pipe. This definition of critical velocity falls between the lower and upper limits of critical velocity defined by the Oroskar and Turian single-phase flow model. Therefore, the experimental data is compatible with the definition of critical velocity used in the Oroskar and Turian single-phase flow model. In this section, the predictions of the proposed model is compared with the observed critical velocities of these two experimental studies. The reported experimental database includes critical velocities obtained using 0.05- and 0.1-m pipe diameters in intermittent and stratified flow regimes. For each pipe size and flow regime, the effect of particle size and particle concentration is investigated. Table 1 shows the operating conditions of the experimental database.

The reason that the proposed model is only validated with low particle concentration range of operating condition is the limited number of experimental data for particles in gas and liquid. The only experimental data available in the literature which is consistent with the definition of critical velocity in the single-phase flow model has been reported for 0.01 and 0.1% of particle volume concentration. On having experimental data for higher concentrations, the validity of the proposed model for higher particle concentration in multiphase flow may be examined. In other words, the proposed model does not impose any restrictions to be used for higher particle concentrations.

General Behavior

Previous experimental studies show that as superficial liquid velocity increases, the required superficial gas velocity to move the particle continuously in the pipe decreases.[5,6,26–28] Figures 2 and 3 compare predictions of various multiphase flow particle transport models with the experimental data reported in literature.[5,6] The uncertainty of the experimental data is discussed in the experimental studies. In the following plots as logarithmic scales is used to present the comparison of experimental data with the models, the uncertainty bars could not be depicted well and as a result, they are not included.

Among the particle transport in multiphase flow models available in the literature, Angelsen et al.,[9] Stevenson and Thorpe model,[17] and Ibarra et al. model[4] are developed for

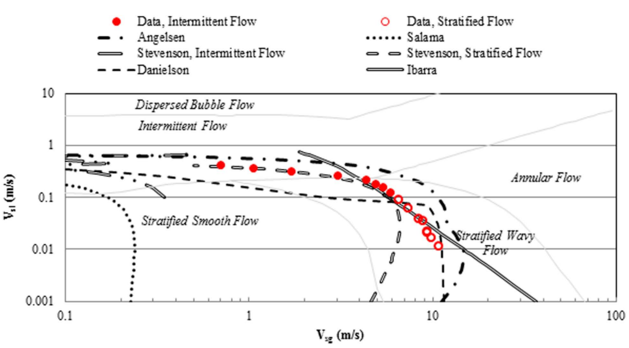

Figure 2. Comparison of critical velocity predicted by existing multiphase models available in the literature with experimental data reported[5,6] air–water flow, pipe diameter: 0.05 m, particle diameter: 300 µm, particle volume concentration: 0.01%.

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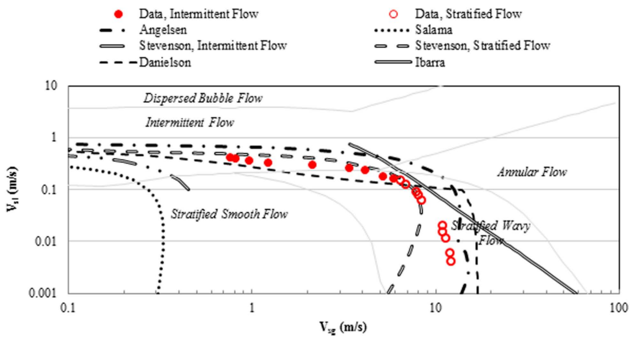

Figure 3. Comparison of critical velocity predicted by existing multiphase models available in the literature with experimental data reported[5,6] air–water flow, pipe diameter: 0.1 m, particle diameter: 300 μm, particle volume concentration: 0.01%.

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stratified flow while Danielson model[18] (Danielson T, Private Communication) and Salama[10] model are independent of multiphase flow regime. Table 2 compares the definition of critical velocities in these studies.

In Figures 2 and 3, all the models are plotted for a wide range of superficial liquid and gas velocities regardless of the multiphase flow regime for which they were developed to compare their general behavior for the same operating conditions. Figure 2 compares the proposed models for the 0.05-m pipe diameter. This figure depicts that, among the above models, the Salama model and Stevenson et al. model for intermittent flow do not follow the trend or the values of experimental data. Surprisingly, the Stevenson and Thorpe model for stratified flow predicts critical velocity well for high superficial liquid and low superficial gas velocities (intermittent flow regime), but as superficial liquid velocity decreases and multiphase phase flow regime moves toward stratified flow, it does not follow the trend of the experimental data. This behavior is even more obvious for very low liquid velocities. Danielson's model under predicts critical velocity in the intermittent flow regime. The Angelsen et al. model follows the trend of data both in intermittent and stratified flow regimes but over predicts critical velocity in both flow regimes. Moreover, based on Figure 2, it seems that this model does not have a consistent trend at lower superficial liquid velocities. Finally, the Ibarra et al. model predicts critical velocity fairly well in stratified flow for 0.05-m pipe. Figure 3 compares the models and experimental data for a pipe diameter of 0.1 m. As Figure 3 shows, the general trend of all the models are more or less the same except for Ibarra's model that significantly over predicts critical velocity

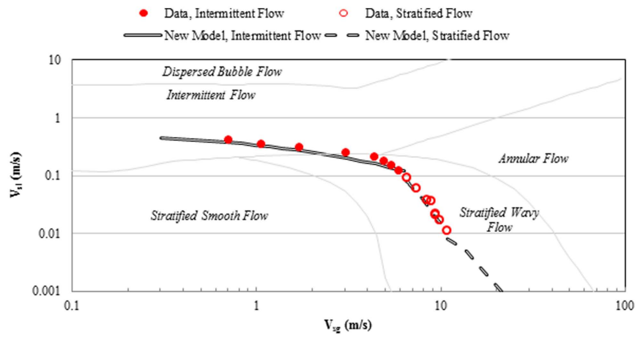

Figure 4. Comparison of critical velocity predicted by the new model with experimental data reported[5,6] air–water flow, pipe diameter: 0.05 m, particle diameter: 300 μm, particle volume concentration: 0.01%.

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in stratified flow when pipe diameter increases. Next, the proposed model is compared with experimental data.

Figures 4 and 5 compare critical velocities reported in the experimental studies in both intermittent and stratified flow regimes with predictions of the proposed model for pipe diameters of 0.05 and 0.1 m, respectively. It is clear that the model is able to predict critical velocity well for a wider range of superficial liquid and gas velocities in both 0.05- and 0.1-m pipe diameter.

Particle Concentration Effect

Previous experimental studies[4–6,19,26–28] demonstrated that increasing particle concentration increases critical velocity.

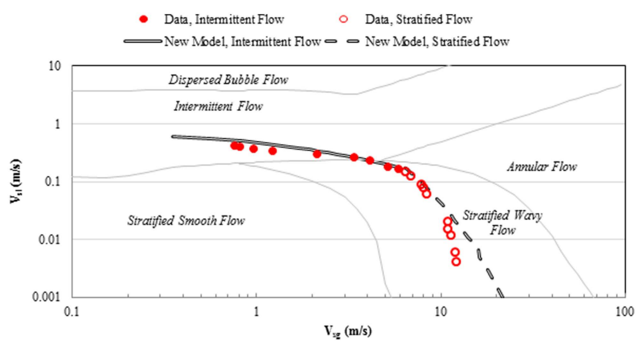

Figure 5. Comparison of critical velocity predicted by the new model with experimental data reported[5,6] air–water flow, pipe diameter: 0.1 m, particle diameter: 300 μm, particle volume concentration: 0.01%.

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Table 2. Definition of Critical Velocities in Previously Proposed Models

Model	Critical Velocity Definition
Angelsen et al.[9]	Velocity required to prevent a sand bed from forming
Salama[10]	Velocity required to fully suspend sand particles
Stevenson et al. and Stevenson and Thorpe[16,17]	Velocity required to prevent a stationary sand bed from forming
Danielson[18]	Velocity required to prevent a sand bed from forming
Ibarra et al.[4]	Velocity required to prevent a stationary sand bed from forming

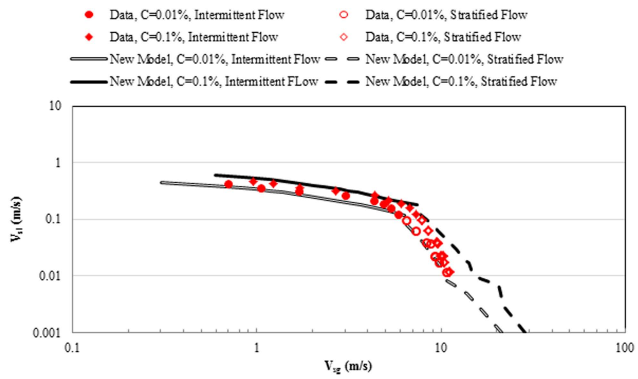

Figure 6. Comparison of critical velocity predicted by the new model with experimental data reported[5,6] air–water flow, pipe diameter: 0.05 m, particle diameter: 300 μm.

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Figures 6 and 7 compare the experimental data and the proposed model predictions for a particle volume concentration of $C = 0.01\%$ and $C = 0.1\%$ in 0.05- and 0.1-m pipe diameter, respectively. The figures reveal that the new model can capture the effect of particle concentration with increases in critical velocity as particle concentration increases. There is good agreement between the experimental data and the predictions for both pipe sizes, although, for the 0.1-m pipe diameter, the new model slightly over predicts critical velocity. As all the models available in the literature (except for Ibarra et al.) are independent of particle concentration, they are not able to capture its effect, and they predict the same trend and values with changing particle concentrations (see Figures 2 and 3). By increasing particle concentration, the Ibarra et al. model predicts slightly higher critical velocity values, but it still over predicts critical velocity significantly for stratified flow in 0.1-m pipe.

A comparison of change in critical velocity predicted by the proposed model with the change in experimental data depicted in Figures 6 and 7 shows that the proposed model over-estimates the effect of particle concentration on critical velocity. This might be explained by considering the fact

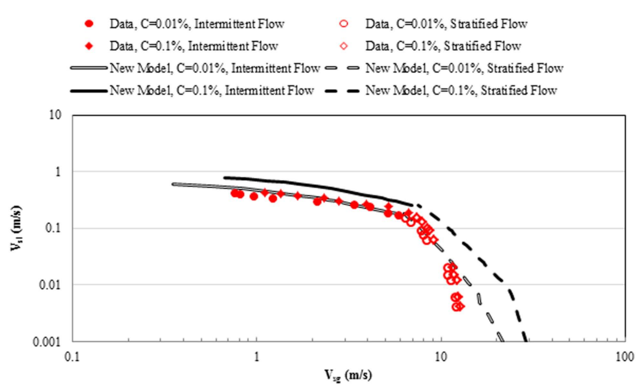

Figure 7. Comparison of critical velocity predicted by the new model with experimental data reported[5,6] air–water flow, pipe diameter: 0.1 m, particle diameter: 300 μm.

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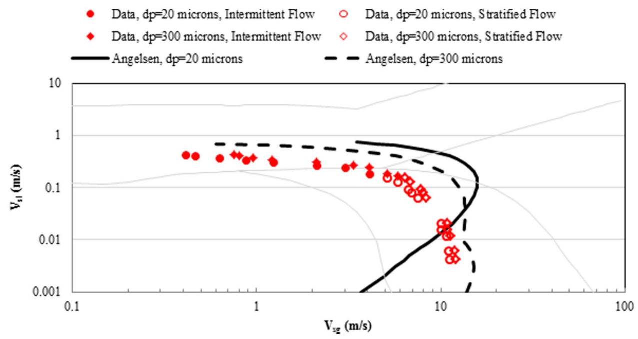

Figure 8. Comparison of critical velocity predicted by Angelsen et al. model with experimental data reported[5,6] air–water flow, pipe diameter: 0.1 m, particle volume concentration: 0.01%.

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that the Oroskar and Turian model is mainly developed for higher particle concentrations (slurries). Therefore, the effect of particle concentration on critical velocity may be modeled well using this model for lower particle concentrations. The authors would like to emphasize that particle concentration has no direct influence on the multiphase flow part of the proposed model. The new model predicts higher critical velocity values for multiphase flow because the critical velocity in the single-phase flow part of the model increases by increasing particle concentration. For example, when particle concentration increases from 0.01 to 0.1%, the Oroskar and Turian model predicts a higher value for critical velocity in single-phase flow. As a result, in the multiphase flow part of the model, at constant superficial liquid velocity, higher actual liquid velocity is required to meet the increase in single-phase flow part of the model. Higher actual liquid velocity at constant superficial liquid velocity corresponds to smaller liquid hold up which can be obtained at higher superficial gas velocity.

Particle Size Effect

Najmi et al.[5,6] reported that critical velocity increases by increasing particle size for the sizes examined in their experimental study. The single-phase Oroskar and Turian model

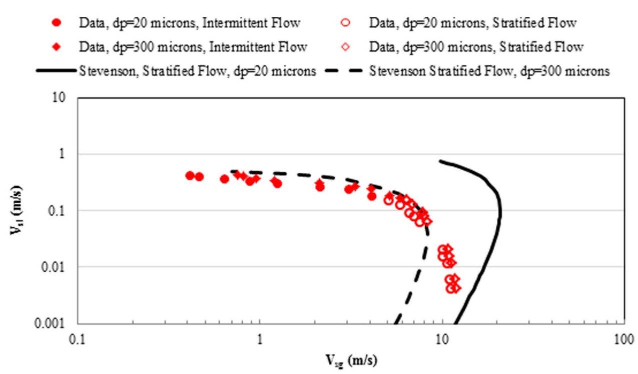

Figure 9. Comparison of critical velocity predicted by Stevenson and Thorpe stratified flow model with experimental data reported[5,6] air–water flow, pipe diameter: 0.1 m, particle volume concentration: 0.01%.

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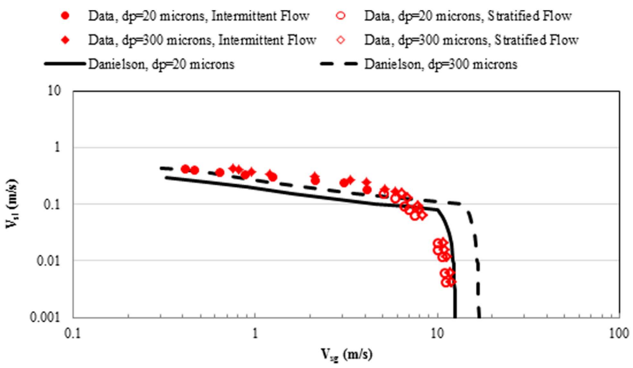

Figure 10. Comparison of critical velocity predicted by Danielson model with experimental data reported[5,6] air–water flow, pipe diameter: 0.1 m, particle volume concentration: 0.01%.

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(which is developed for particles of a fixed size and density) also predicts higher values for critical velocity as particle size increases. In this section, the effect of particle size on critical velocity predicted by the models in literature and the proposed model in multiphase flow is investigated. As the Angelsen et al. model, Stevenson and Thorpe model for stratified flow and Danielson model follow the trend of experimental data better than other models (see Figures 2 and 3), only these models are considered in this section. As similar behavior is predicted by these models, comparison is only shown for larger pipe diameter. Figures 8–10 compare the behavior of these models and experimental data for two different particle sizes (with 20 and 300 μm average diameters) for 0.1-m pipe diameter. As these figures depict, experimental data shows that critical velocity increases by increasing particle size for the particle sizes examined. Angelsen et al. model and Stevenson and Thorpe model for stratified flow cannot capture the effect of particle size, and predict lower values for critical velocity with increasing particle sizes. Figures 8 and 9 also reveal that these models have limitations for smaller particle sizes and they do not converge in intermittent flow regime (for small values of superficial gas velocities, the model gives nonphysical values and cannot cover the whole range of superficial liquid and gas velocities examined in the experimental studies). This behavior is not surprising as these models are developed for stratified flow. But even in stratified flow, they dramatically over predict or under predict critical velocity for the smaller particle size. Figure 10 shows the change in Danielson model predictions for the two particle sizes considered and compares it with experimental data. As this figure shows, Danielson model captures the effect of particle size and predicts higher critical values by increasing particle size. Although this model follows the trend of data it under predicts critical velocity in intermittent flow and over predict it in stratified flow regime, for both particle sizes. It is worth mentioning that these models behave similarly in 0.05-m pipe diameter.

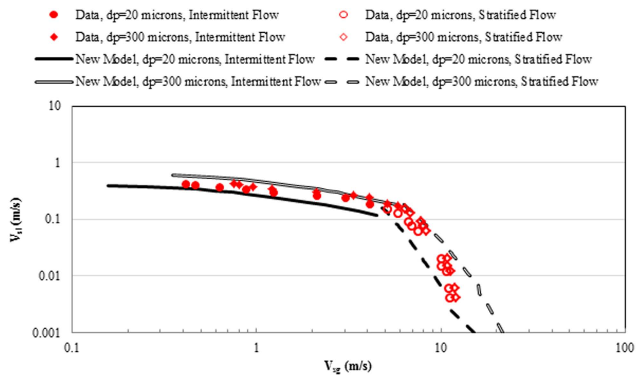

Figure 12. Comparison of critical velocity predicted by the new model with experimental data reported[5,6] air–water flow, pipe diameter: 0.1 m, particle volume concentration: 0.01%.

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Figures 11 and 12 show how the predictions of the proposed model changes for two different particle sizes and compare it with experimental data for particle concentration of 0.01% in 0.05- and 0.1-m pipe diameters, respectively. These figures depict that the proposed model captures the

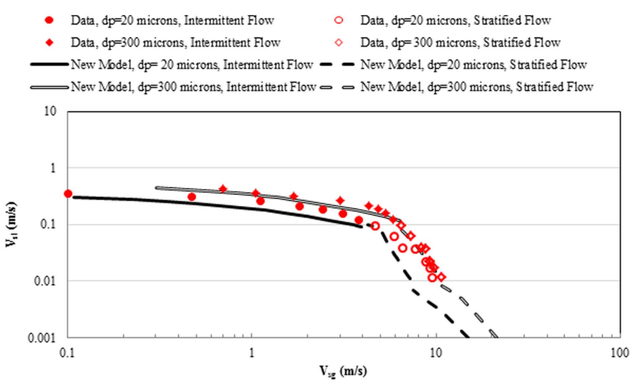

Figure 11. Comparison of critical velocity predicted by the new model with experimental data reported[5,6] air–water flow, pipe diameter: 0.05 m, particle volume concentration: 0.01%.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

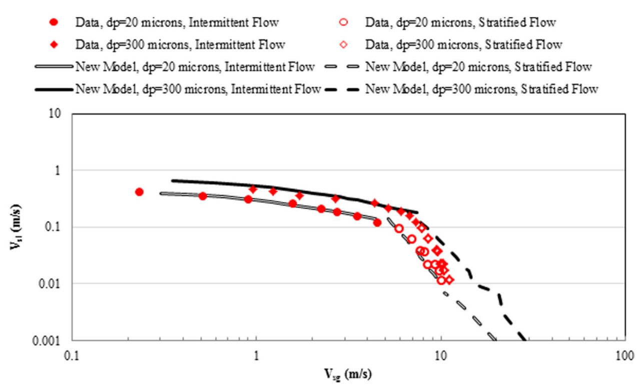

Figure 13. Comparison of critical velocity predicted by the new model with experimental data reported [5,6] air–water flow, pipe diameter: 0.05 m, particle volume concentration: 0.1%.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

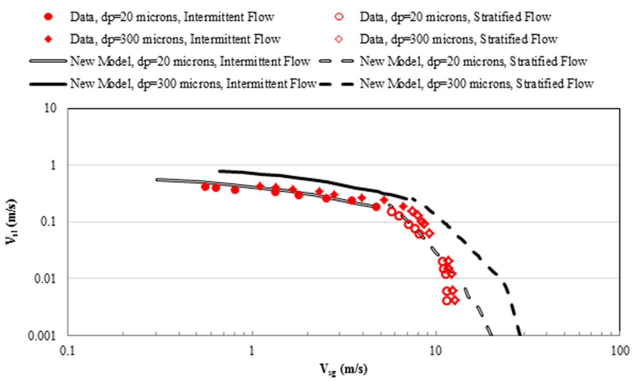

Figure 14. Comparison of critical velocity predicted by the new model with experimental data reported[5,6] air–water flow, pipe diameter: 0.1 m, particle volume concentration: 0.1%.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

effect of particle size and predicts higher values for critical velocity as particle size increases. Furthermore, there is good agreement between the experimental data and model predictions for both pipe sizes even for the small particle size (20 μm average diameter). This agreement is of great importance as the database that Oroskar and Turian used to develop their model did not include experimental data for particles smaller than 100 μm. However, good agreement between the experimental data obtained in single-phase liquid flow using 20 μm silica sand and the Oroskar and Turian model prediction has been previously reported in the literature.[23] The proposed model slightly under predicts critical velocity for intermittent flow for the small particle size in both pipe sizes. Figures 13 and 14 show the same comparison for higher particle concentration ($C = 0.1\%$). As these figures reveal, even at higher concentration, there is good agreement between the experimental data and model predictions. The proposed model captures the effect of particle size and particle concentration in both pipe sizes. For higher particle concentrations and for the large particle size (300 μm average diameter), the proposed model over predicts critical velocity in stratified flow regime, but for the small particle size and high particle concentrations the proposed model predicts both the trend and values of critical velocity well for both pipe sizes. The experimental data reported by Najmi et al.[5,6] also includes particle diameter of 150 μm. As the critical velocity difference between 300 μm particle and 150 μm particle is not as significant as 300 μm particle and 20 μm particle, comparison of the proposed model with experimental data for medium particle size is not included in the following plots. However, the comparison for this particle size reveals that the proposed model predicts critical velocity well for 0.01 and 0.1% of particle concentration in both small and large pipe diameters. Obviously, as the Oroskar and Turian model was developed for particles of fixed size and density, the proposed sand transport model in multiphase flows is only applicable to particles of fixed size and density.

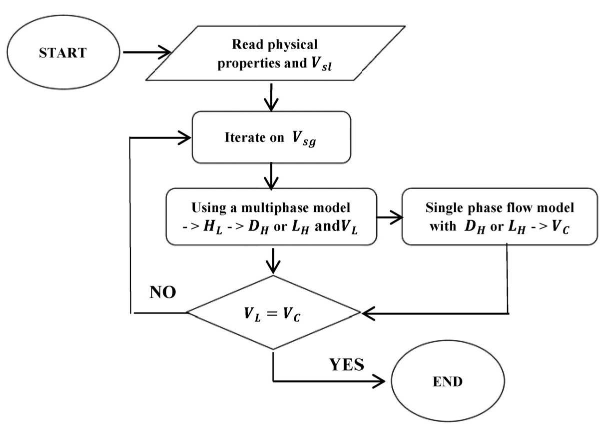

Figure 15. The updated algorithm for calculating critical velocity using hydraulic pipe diameter or liquid height as a length scale.

Investigation of Applying Other Length Scales

Another approach to extend a single-phase particle transport model to a multiphase flow one is using other length scales instead of pipe diameter. In this section, the possibility of using hydraulic pipe diameter (D_H) and liquid height (L_H) to extend the Oroskar and Turian model to multiphase flow regime is investigated. In the plots presented later, these three length scales (including pipe diameter which was previously used in developing the new model) are defined as D, D_H, and L_H which represent pipe diameter, hydraulic pipe diameter, and liquid height, respectively. When the two latter length scales are used for predictions, the algorithm to calculate critical velocity should be modified to incorporate the hydraulic pipe diameter or liquid height in the pipe in place of pipe diameter in the single-phase flow model. Figure 15 gives the updated algorithm for using length scales other than pipe diameter. As can be seen in Figure 15, in every iteration of gas superficial velocity, actual liquid velocity obtained using multiphase flow part of the model is compared with the Oroskar and Turian critical velocity predictions obtained using other length scales instead of pipe diameter to calculate the corresponding single-phase flow critical velocity.

The comparison of using other length scales with experimental data and the proposed model reveals that using hydraulic pipe diameter and liquid height instead of pipe

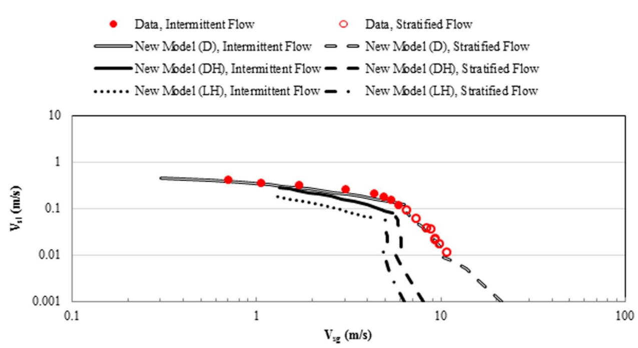

Figure 16. Comparison of using different length scales to extend the Oroskar and Turian model to multiphase flow with experimental data reported[5,6] air–water flow, pipe diameter: 0.05 m, particle size: 300 μm, particle volume concentration: 0.01%.

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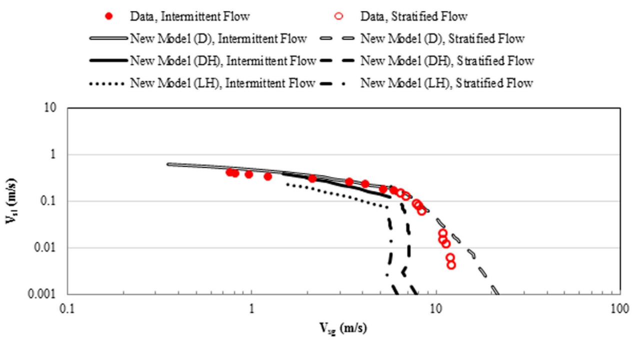

Figure 17. Comparison of using different length scales to extend the Oroskar and Turian model to multiphase flow with experimental data reported[5,6] air–water flow, pipe diameter: 0.1 m, particle size: 300 μm, particle volume concentration: 0.01%.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

diameter in the Oroskar and Turian model consistently results in under prediction of critical velocity. Because a similar behavior is observed for all particle concentrations, particle sizes, and pipe diameters, here we only present comparisons of the predictions with experimental data for one particle concentration and particle size in both pipe diameters. The comparison (see Figures 16 and 17) reveal that pipe diameter is the appropriate length scale to extend the single-phase Oroskar and Turian model to multiphase flow.

Error Analysis

In this section, the discrepancy of model predictions with the experimental data is investigated for a selected subset of the models for particle transport in multiphase flow from the literature and the proposed model. A comparison of the models from the literature with experimental data depicted in Figures 2 and 3 shows that the Angelsen et al., Stevenson and Thorpe stratified flow, and Danielson models follow the trend of experimental data better than other models. Therefore, these models are the ones included in our subset and are the ones compared with the proposed model here. It is worth noting that, Danielson's model is developed for both intermittent and stratified flow. Although

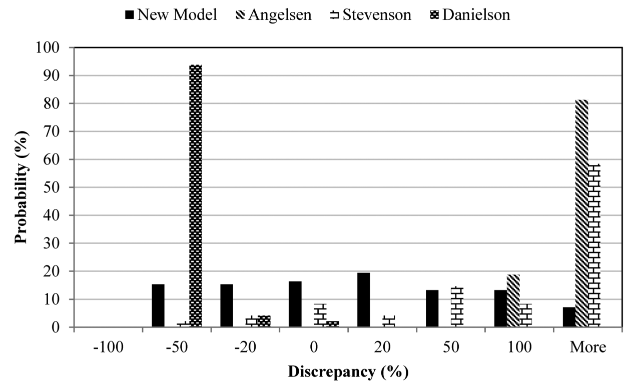

Figure 19. Discrepancy distribution of the selected models in intermittent flow regime.

comparisons of the Angelsen et al. model and the Stevenson and Thorpe stratified model with experimental data show that they follow the trend of data in the intermittent flow regime, it should be considered that these models were developed for stratified flow. However, to have a general investigation, the three selected models and the proposed model are analyzed for the entire range of operating conditions in this section. To calculate the discrepancy of the models, the minimum superficial gas velocities predicted by the models are compared with the corresponding experimental values for superficial gas velocity at a fixed superficial liquid velocity

$$\text{Discrepancy}\ (\%)=\frac{V_{sg(m)}-V_{sg(e)}}{V_{sg(e)}}\times 100$$

where $V_{sg(m)}$ and $V_{sg(e)}$ represent superficial gas velocities predicted by the models and obtained experimentally, respectively. Based on the above definition, positive values for calculated discrepancy are the regions where the models over predict critical velocity while negative values represent under predication of the models. The probability of a model having discrepancies between a specified range is calculated by dividing the number of data points within each specified range by the total number of data points.

Figure 18 shows the discrepancy distribution of the Angelsen et al., Stevenson and Thorpe stratified, Danielson and the proposed model for the data reported in the previous experimental studies.[5,6] There are 211 data points for two pipe sizes (0.05 and 0.1 m), three particle sizes (300, 150, and 20 μm), two particle volume concentrations (0.01 and 0.1%) in both

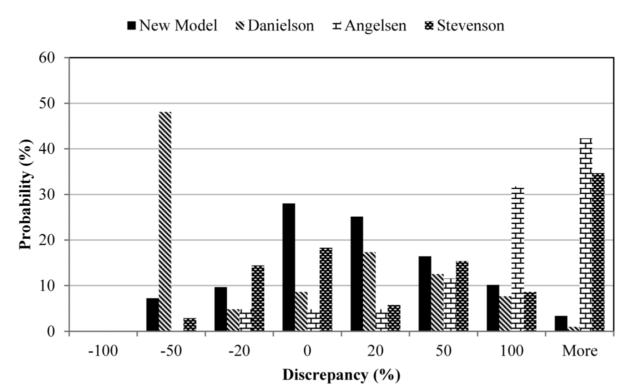

Figure 18. Discrepancy distribution of the selected models for the whole range of operating conditions reported in experimental studies.[5,6]

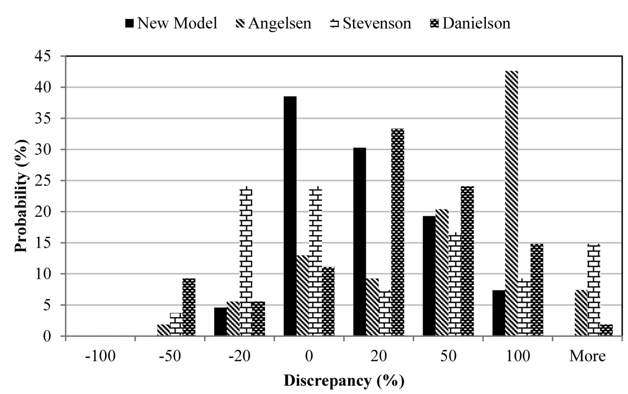

Figure 20. Discrepancy distribution of the selected models in stratified flow regime.

Table 3. **PPMC Values Calculated for the Selected Models in Intermittent and Stratified Flow Regimes**

	Pipe Size		Particle Size		Particle Concentration	
	Intermittent	Stratified	Intermittent	Stratified	Intermittent	Stratified
Proposed Model	0.47	0.4	0.38	0.46	0.52	0.51
Angelsen et al.[9]	0.02	0.14	−0.39	−0.04	NA	NA
Stevenson and Thorpe[17]	−0.014	0.07	−0.45	−0.68	NA	NA
Danielson[18] (Danielson T, Private Communication)	0.46	0.33	0.11	0.21	NA	NA

intermittent and stratified flow regimes (including low liquid loading). The proposed model is compared with all 211 data points. This figure depicts that the proposed model predicts 55% of experimental data within ±20% of accuracy and 80% of data within ±50% of accuracy. The graph also reveals that the predictions of the proposed model predictions are usually on the safe side (the model over predicts critical velocity). The models from the literature are independent of particle concentration. Therefore, the comparisons are only performed for the low particle concentration data where they show smaller deviation from experimental data, and the probability plots in Figure 18 are obtained using only 104 data points. Figure 18 shows that Danielson's model tends to under predict critical velocity for the operating conditions examined where the Angelsen et al. and Stevenson and Thorpe models usually over predict critical velocity.

Figure 19 presents the discrepancy distributions of the selected models for intermittent flow. In this figure, the proposed model is compared with experimental data at both particle concentrations while other models are only compared with the data at the lower particle concentration. It seems that the proposed model predicts critical velocity fairly well and there is a relatively even distribution of discrepancy between ±50% range while other models extremely over predict or under predict critical velocity. As aforementioned, poor functionality of the Angelsen et al. and Stevenson and Thorpe models in intermittent flow is not surprising because they were not developed for the intermittent flow regime, but Danielson's model which is developed for both intermittent and stratified flow is not able to predict critical velocity in this flow regime. It seems that although the proposed model can predict critical velocity fairly well in intermittent flow there is still room to further improve velocity predictions. The discrepancy analysis in this flow regime also suggests that caution should be taken in using logarithmic scales. Although previous plots (such as Figures 11 and 14) implied that the other models can follow the trend of data well in intermittent flow regime, discrepancy analysis reveals that the logarithmic axis scales in these graphs masks the relatively significant discrepancies in the model's predictions.

Figure 20 depicts the discrepancy distribution of selected models in stratified flow. It is clear that the proposed model improves prediction of critical velocity to a great extent. This model is able to predict 70% of the critical velocities reported in the experimental studies within ±20% of accuracy, which is a significant improvement when compared with the existing models in the literature for stratified flow regime.

To investigate discrepancy dependency of the models to physical parameters, the Pearson product-moment correlation coefficient (PPMC) of the discrepancy of selected models for the physical parameters investigated in this study is calculated. PPMC is a scalar value which represents linear dependency of input and output of a function. PPMC values ranges from −1 to 1, where 1 represents total positive correlation, 0 represents no dependence, and −1 represents total negative correlation. In other words, PPMC = 1 means there is an exact increasing linear correlation between input and output of a function. Generally, $\mathrm{PPMC} < |0.35|$ and $\mathrm{PPMC} > |0.68|$ represent weak and strong linear dependency, respectively.[29] Based on the obtained values for each flow regime (Table 3), it seems that none of the physical parameters in the models correlates linearly with the discrepancy of the models. Conversely, moderate values of PPMC for the proposed model confirm that there is still room for further improvement of a multiphase flow model to predict critical particle transport velocity.

Conclusion

A new model to predict minimum required velocities to successfully transport particle in multiphase water–air flows is suggested. The new model is the first model that can be applied to both intermittent and stratified flow regimes and also takes into account the effect of particle concentration. Comparison of the proposed model with previously reported data in the literature shows that the model is able to predict critical velocity well for a wide range of operating conditions including different pipe sizes, particle sizes, particle concentrations, and multiphase flow regimes. As previously reported in the experimental studies, the proposed model predicts higher critical velocities by increasing pipe size, particle size, and particle concentration for the conditions investigated. The error analysis of the proposed model and comparison with other multiphase flow models in the literature show that the new model improves predication of critical velocity significantly in both intermittent and stratified flow regimes, but performs better for stratified flow than intermittent flow. The PPMC of the discrepancy of the proposed model reveals that although this model is not a strong linear function of the physical parameters investigated in this study, but there is still room for further improvement of a multiphase flow model to predict critical particle transport velocity. Along with the new model, the possibility of applying other multiphase flow length scales to extend the single-phase model used in this study to multiphase flow is also investigated. Results show that pipe diameter is the best length scale that fits experimental data to extend the Oroskar and Turian single-phase critical velocity model to multiphase air–water flow.

Acknowledgments

The authors greatly acknowledge Tulsa University Sand Management Projects (TUSMP) members for their financial support of this study. Without their financial support, this study would not have been possible. Kamyar Najmi would also like to thank Prof. Ovadia Shoham for his great support, Dr. Gene Kouba from Chevron Energy Technology for his useful comments and also the University of Tulsa graduate school for awarding the Bellwether Ph.D. fellowship during this study.

Notation

A = cross-sectional area inside the pipe
C = particle volume concentration
C_1 = coefficient
C_e = coefficient
d_p = particle diameter
D = pipe diameter
D_H = hydraulic pipe diameter
F_E = entrainment fraction of liquid in gas core
Fr = Froude number
f = friction factor
g = gravitational acceleration
H_L = liquid hold up
L_H = liquid height
$\overline{L_H}$ = liquid film height with flat interface
l = length
p = pressure
Re = Reynolds number
S = perimeter
V = velocity
V_0 = gas bubble rise velocity relative to liquid
We = Weber number
z = axial length
α = liquid hold up to gas void fraction ratio
β = exponent
Θ = wetted wall fraction
θ = inclination angle
μ = dynamic viscosity
v = slug frequency
ρ = density
σ = surface tension
τ = shear stress
x = eddy fraction

Subscripts

C = critical
CO = gas core
CD = chord
F = liquid film
G = gas
I = interface
L = liquid
p = particle
S = slug
sg = superficial gas
sl = superficial liquid
T = translational
t = transition
U = slug unit
W = wall

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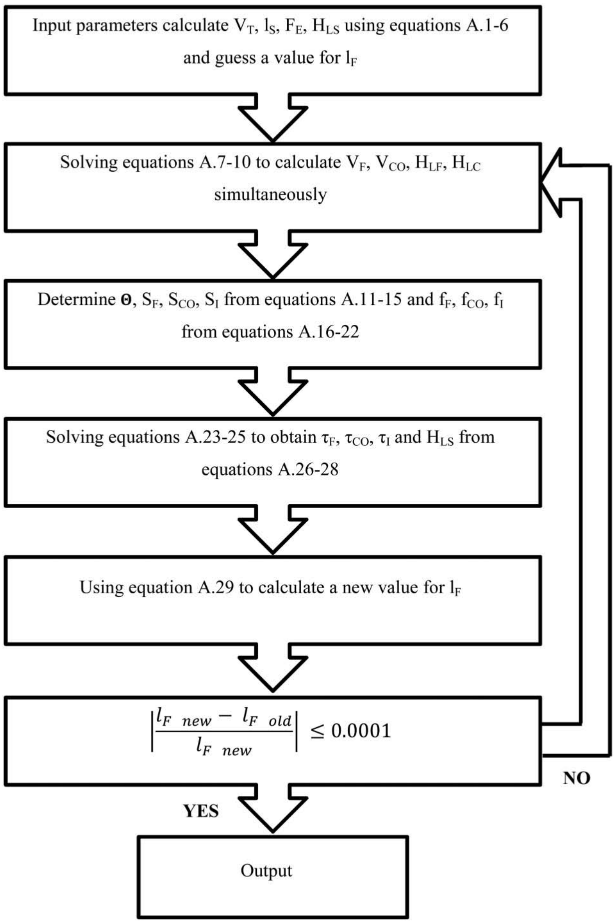
Appendix A

The flowchart and governing equations in the Zhang et al.[20] unified model



$$V_T = C_S V_S + V_D \tag{A1}$$

$$V_D = 0.54\sqrt{gD}\cos\theta + 0.35\sqrt{gD}\sin\theta \tag{A2}$$

$$l_S = \left(32.0\cos^2\theta + 16.0\sin^2\theta\right)D \tag{A3}$$

$$\frac{F_E}{1-F_E} = 0.0003 We_{sg}^{1.8} Fr_{sg}^{-0.92} Re_{sl}^{0.7} Re_{sg}^{-1.24}\left(\frac{\rho_L}{\rho_G}\right)^{0.38}\left(\frac{\mu_L}{\mu_G}\right)^{0.97} \tag{A4}$$

$$We_{sg} = \frac{\rho_G V_{sg}^2 D}{\sigma} \tag{A5}$$

$$\rho_s = \rho_L H_{LS} + \rho_G (1 - H_{LS}) \tag{A6}$$

$$H_{LS}(V_T - V_S) = H_{LF}(V_T - V_F) + H_{LCO}(V_T - V_{CO}) \tag{A7}$$

$$(1 - H_{LS})(V_T - V_S) = (1 - H_{LF} - H_{LCO})(V_T - V_{CO}) \tag{A8}$$

$$l_U V_{SL} = l_S H_{LS} V_S + l_F (1 - H_{LF} - H_{LCO}) V_{CO} \tag{A9}$$

$$F_E = \frac{H_{LCO} V_{CO}}{H_{LF} V_F + H_{LCO} V_{CO}} \tag{A10}$$

$$\Theta = \Theta_0\left(\frac{\sigma_{water}}{\sigma}\right)^{0.15} + \frac{\rho_G}{\rho_L - \rho_G}\frac{1}{\cos\theta}\left(\frac{\rho_L V_{sl}^2 D}{\sigma}\right)^{0.25}\left(\frac{V_{sg}^2}{(1-H_{LF})^2 gD}\right)^{0.8} \tag{A11}$$

$$S_I = \frac{S_F(A_{CD} - A_F) + S_{CD} A_F}{A_{CD}} \tag{A12}$$

$$S_F = \pi D \Theta \tag{A13}$$

$$S_{CD} = D\sin(\pi\Theta) \tag{A14}$$

$$A_{CD} = \frac{D^2}{4}\left(\pi\Theta - \frac{\sin(2\pi\Theta)}{2}\right) \tag{A15}$$

$$f = C_1 Re^{-n} \tag{A16}$$

$$Re_F = \frac{4 A_F V_F \rho_L}{S_F \mu_L} \tag{A17}$$

$$Re_C = \frac{4 A_{CO} V_{CO} \rho_G}{(S_{CO} + S_I)\mu_G} \tag{A18}$$

$$A_F = H_{LF} A \tag{A19}$$

$$A_{CO} = (1 - H_{LF}) A \tag{A20}$$

$$f_I = f_{CO}\left\{1 + 15\left(\frac{H_L}{D}\right)^{0.5}\left(\frac{V_{sg}}{V_{sg,t}} - 1\right)\right\} \tag{A21}$$

$$V_{sg,t} = 5\left(\frac{1.24}{\rho_g}\right)^{0.5} \tag{A22}$$

$$\tau_F = f_F \frac{\rho_L V_F^2}{2} \tag{A23}$$

$$\tau_{CO} = f_{CO}\frac{\rho_G V_{CO}^2}{2} \tag{A24}$$

$$\frac{\tau_I = f_I \rho_{CO}(V_{CO} - V_F)|V_{CO} - V_F|}{2} \tag{A25}$$

$$H_{LS} = \frac{1}{1 + \frac{T_{sm}}{3.16[(\rho_L - \rho_G)g\sigma]^{0.5}}} \tag{A26}$$

$$T_{sm} = \frac{1}{C_e}\left(\frac{f_s}{2}\rho_s V_s^2 + \frac{D}{4}\frac{\rho_L H_{LF}(V_T - V_F)(V_S - V_F)}{l_S} + \frac{d}{4}\frac{\rho_C H_{LF}(V_T - V_{CO})(V_S - V_{CO})}{l_S}\right) \tag{A27}$$

$$\frac{C_e = 2.5 - |\sin\theta|}{2} \tag{A28}$$

$$\frac{\rho_L(V_T - V_F)(V_S - V_F) - \rho_{CO}(V_T - V_{CO})(V_S - V_{CO})}{l_F} - \frac{\tau_F S_F}{H_{LF} A} + \frac{\tau_{CO} S_{CO}}{(1 - H_{LF})A} + \tau_I S_I\left(\frac{1}{H_{LF} A} + \frac{1}{(1 - H_{LF})A}\right) - (\rho_L - \rho_{CO}) g \sin\theta = 0 \tag{A29}$$

Appendix B

Fan[21] defined α as the ratio of liquid hold up to gas void fraction and stated that the combined momentum equation for low liquid loading flows can be simplified to

$$\alpha=\left(\frac{V_{sl}}{V_{sg}}\right)\left(\frac{f_L\rho_L S_L}{f_I\rho_G S_I}\right)^{0.5} \quad \text{(B1)}$$

He stated that although Eq. B1 is not used to calculate α it shows the importance of this parameter. In the above equation, superficial liquid and gas velocities are given as operational conditions. The gas and liquid densities are generally known or can easily be calculated. Therefore, the liquid-wall friction factor, interfacial friction factor, wetted wall perimeter, and interfacial perimeter are unknowns and need to be determined using closure relationships. He defined the closure relationships as follows

Wetted wall fraction

$$\Theta=\left\{0.57H_L^{0.345}+0.0637\ Fr_L^{0.68}\left(\frac{V_{sg}}{V_{sg,c}}\right)^{0.68}\right\}\left(\frac{\sigma_W}{\sigma_L}\right)^{0.15}$$

for $0\leq\Theta\leq 0.5$ (B2a)

$$\Theta=\left\{0.57H_L^{0.345}+0.0637\ Fr_L^{0.68}\left(\frac{V_{sg}}{V_{sg,c}}\right)^{0.55}\right\}\left(\frac{\sigma_W}{\sigma_L}\right)^{0.15}$$

for $0.5\leq\Theta\leq 1.0$

(B2b)

$$S_L=\pi D\Theta \quad \text{(B3)}$$

$$S_G=\pi D(1-\Theta) \quad \text{(B4)}$$

$$V_{sg,c}=5\left(\frac{1.24}{\rho_G}\right)^{0.5} \quad \text{(B5)}$$

Interfacial perimeter

$$S_I=\frac{S_L(A_{CD}-A_L)+S_{CD}A_L}{A_{CD}} \quad \text{(B6)}$$

$$A_{CD}=\frac{D^2}{4}\left(\pi\Theta-\frac{\sin(2\pi\Theta)}{2}\right) \quad \text{(B7)}$$

Gas-wall friction factor

$$f_G=\frac{16}{Re_G} \quad \text{for} \quad Re_G\ \leq 2000 \quad \text{(B8a)}$$

$$f_G=0.046Re_G^{-0.2} \quad \text{for} \quad Re_G\ \geq 2000 \quad \text{(B8b)}$$

$$Re_G=\frac{\rho_G V_G D_G}{\mu_G} \quad \text{(B9)}$$

$$D_G=\frac{4\ A_g}{S_g+S_I} \quad \text{(B10)}$$

Liquid-wall friction factor

$$f_L=\frac{8}{Re_L} \quad \text{for} \quad Re_L\ \leq 1000 \quad \text{(B11a)}$$

$$f_L=0.0709Re_L^{-0.2666} \quad \text{for} \quad 1000\leq Re_L\ \leq 25,000 \quad \text{(B11b)}$$

$$Re_L=\frac{\rho_L V_L D_L}{\mu_L} \quad \text{(B12)}$$

$$D_L=\frac{4\ A_L}{S_L} \quad \text{(B13)}$$

Interfacial friction factor

$$\frac{f_I}{f_G}=1 \quad \text{for} \quad V_{sg}\leq V_{sg,c} \quad \text{(B14a)}$$

$$\frac{f_I}{f_G}=1+21\left(\frac{\overline{H_L}}{D}\right)^{0.72}\left(\frac{V_{sg}}{V_{sg,c}}-1\right)^{0.8} \quad \text{for} \quad V_{sg}\geq V_{sg,c}$$

(B14b)

$$\overline{H_L}=\frac{2A_L}{S_L+S_I} \quad \text{(B15)}$$

$$V_{sg,c}=5\left(\frac{1.24}{\rho_G}\right)^{0.5} \quad \text{(B16)}$$

Having the above closure relationships, the combined momentum equation can be solved iteratively for α. On convergence of α, all the variables are calculated for that flow conditions.

Manuscript received Dec. 1, 2014, and revision received Feb. 24, 2015.